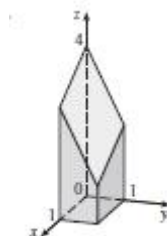


부록 E 해답

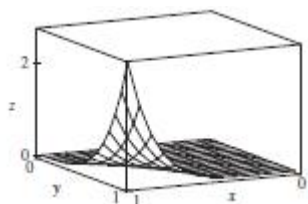
12장

연습문제 12.1

01. (a) 288 (b) 144 02. (a) 0.990 (b) 1.151 03. 248 04. 60
 05. 3 06. 222 07. $32(e^4 - 1)$ 08. 18 09. $\frac{21}{2} \ln 2$ 10. $\frac{31}{30}$



11. $9 \ln 2$ 12. $\frac{1}{2}(\sqrt{3} - 1) - \frac{1}{12}\pi$ 13. $\frac{1}{2}e^{-6} + \frac{5}{2}$ 14.
 15. 51 16. $\frac{166}{27}$ 17. 2 18. $\frac{64}{3}$



19. $21e - 57$; 20. $\frac{5}{6}$

$$\begin{aligned} 21. \iint_R k dA &= \lim_{m, n \rightarrow \infty} \sum_{i=1}^m \sum_{j=1}^n f(x_{ij}^*, y_{ij}^*) \Delta A = \lim_{m, n \rightarrow \infty} k \sum_{i=1}^m \sum_{j=1}^n \Delta A \\ &= \lim_{m, n \rightarrow \infty} k(b-a)(d-c) = k(b-a)(d-c) \end{aligned}$$

22. 생략 23. 0

24. 푸비니 정리를 적용할 수 없다. 피적분함수가 원점에서 무한대인 불연속이다.

연습문제 12.2

01. 32 02. $\frac{3}{10}$ 03. $\frac{1}{3} \sin 1$ 04. $\frac{4}{3}$ 05. π

06. 유형 I: $D = \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq x\}$,

유형 II: $D = \{(x, y) \mid 0 \leq y \leq 1, y \leq x \leq 1\}$; $\frac{1}{3}$

$$07. \int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} y dy dx + \int_1^4 \int_{x-2}^{\sqrt{x}} y dy dx = \int_{-1}^2 \int_{y^2}^{y+2} y dx dy = \frac{9}{4}$$

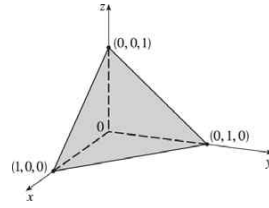
08. $\frac{1}{2}(1 - \cos 1)$ 09. $\frac{11}{3}$ 10. 0 11. $\frac{17}{60}$ 12. $\frac{31}{8}$ 13. 6

14. $\frac{128}{15}$

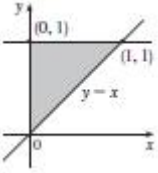
15. $\frac{1}{3}$

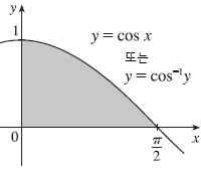
16. $\frac{64}{3}$

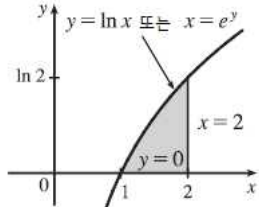
17.



18. $\pi/2$

19.  ; $\int_0^1 \int_x^1 f(x, y) dy dx$

20.  ; $\int_0^1 \int_0^{\cos^{-1} y} f(x, y) dx dy$

21.  ; $\int_0^{\ln 2} \int_{e^y}^2 f(x, y) dx dy$

22. $\frac{1}{6}(e^9 - 1)$

23. $\frac{1}{3} \ln 9$

24. $\frac{1}{3}(2\sqrt{2} - 1)$

25. 1

26. 양쪽 다 유형 II인 영역으로 D를 표현하면 다음과 같다.

$$D = \{(x, y) \mid -1 \leq y \leq 0, -1 \leq x \leq y - y^3\} \cup \{(x, y) \mid 0 \leq y \leq 1, \sqrt{y} - 1 \leq x \leq y - y^3\}$$

$$\begin{aligned} \iint_D y dA &= \int_{-1}^0 \int_{-1}^{y-y^3} y dx dy + \int_0^1 \int_{\sqrt{y}-1}^{y-y^3} y dx dy \\ &= \int_{-1}^0 [xy]_{x=-1}^{x=y-y^3} dy + \int_0^1 [xy]_{x=\sqrt{y}-1}^{x=y-y^3} dy \\ &= \int_{-1}^0 (y^2 - y^4 + y) dy + \int_0^1 (y^2 - y^4 - y^{3/2} + y) dy \\ &= \left[\frac{1}{3}y^3 - \frac{1}{5}y^5 + \frac{1}{2}y^2 \right]_{-1}^0 + \left[\frac{1}{3}y^3 - \frac{1}{5}y^5 - \frac{2}{5}y^{5/2} + \frac{1}{2}y^2 \right]_0^1 \\ &= \left(0 - \frac{11}{30}\right) + \left(\frac{7}{30} - 0\right) = -\frac{2}{15} \end{aligned}$$

27. $0 \leq \iint_D \sqrt{x^3 + y^3} dA \leq \sqrt{2}$

28. $m \leq f(x, y) \leq M, \iint_D m dA \leq \iint_D f(x, y) dA \leq \iint_D M dA$

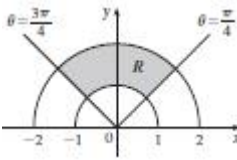
$$\Rightarrow m \iint_D 1 dA \leq \iint_D f(x, y) dA \leq M \iint_D 1 dA$$

$$\Rightarrow mA(D) \leq \iint_D f(x, y) dA \leq MA(D)$$

$$29. 9\pi \quad 30. a^2b + \frac{3}{2}ab^2 \quad 31. \pi a^2b$$

연습문제 12.3

$$01. \int_0^{3\pi/2} \int_0^4 f(r \cos \theta, r \sin \theta) r dr d\theta \quad 02. \int_{-1}^1 \int_0^{(x+1)/2} f(x, y) dy dx$$



$$03. \text{ ; } 3\pi/4 \quad 04. \frac{1250}{3} \quad 05. (\pi/4)(\cos 1 - \cos 9)$$

$$06. \frac{3}{64}\pi^2 \quad 07. \frac{16}{3}\pi \quad 08. \frac{4}{3}\pi a^3 \quad 09. (2\pi/3)[1 - (1/\sqrt{2})]$$

$$10. (8\pi/3)(64 - 24\sqrt{3}) \quad 11. \pi/12 \quad 12. \frac{\pi}{2}(1 - \cos 9) \quad 13. 2\sqrt{2}/3$$

$$14. 37.5\pi \text{ m}^3 \quad 15. \frac{15}{16}$$

$$16. (a) \text{ 각 } a \text{에 대해 } \iint_{D_a} e^{-(x^2+y^2)} dA = \int_0^{2\pi} \int_0^a r e^{-r^2} dr d\theta = 2\pi \left[-\frac{1}{2} e^{-r^2} \right]_0^a = \pi(1 - e^{-a^2}) \text{ 이다.}$$

$$a \rightarrow \infty \text{ 일 때 } e^{-a^2} \rightarrow 0 \text{ 이므로 } \lim_{a \rightarrow \infty} \pi(1 - e^{-a^2}) = \pi \text{ 이다.}$$

$$\text{따라서 } \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} dA = \pi \text{ 이다.}$$

$$(b) \text{ 각 } a \text{에 대해 } \iint_{S_a} e^{-(x^2+y^2)} dA = \int_{-a}^a \int_{-a}^a e^{-x^2} e^{-y^2} dx dy = \left(\int_{-a}^a e^{-x^2} dx \right) \left(\int_{-a}^a e^{-y^2} dy \right) \text{ 이다.}$$

$$(a) \text{로부터 } \pi = \iint_{\mathbb{R}^2} e^{-(x^2+y^2)} dA \text{ 이므로}$$

$$\begin{aligned} \pi &= \lim_{a \rightarrow \infty} \iint_{S_a} e^{-(x^2+y^2)} dA = \lim_{a \rightarrow \infty} \left(\int_{-a}^a e^{-x^2} dx \right) \left(\int_{-a}^a e^{-y^2} dy \right) \\ &= \left(\int_{-\infty}^{\infty} e^{-x^2} dx \right) \left(\int_{-\infty}^{\infty} e^{-y^2} dy \right) \text{ 이다.} \end{aligned}$$

(c) 생략

$$(d) t = \sqrt{2} x \text{로 놓으면 } \int_{-\infty}^{\infty} e^{-x^2} dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2}} \left(e^{-t^2/2} \right) dt \text{ 이다.}$$

$$\text{따라서 } \sqrt{\pi} = \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} e^{-t^2/2} dt \text{ 또는 } \int_{-\infty}^{\infty} e^{-x^2/2} dx = \sqrt{2\pi} \text{ 이다.}$$

17. (a) $\sqrt{\pi}/4$ (b) $\sqrt{\pi}/2$

연습문제 12.4

01. 285 C 02. $42k, \left(2, \frac{85}{28}\right)$ 03. $6, \left(\frac{3}{4}, \frac{3}{2}\right)$ 04. $\frac{8}{15}k, \left(0, \frac{4}{7}\right)$

05. $L/4, (L/2, 16/(9\pi))$ 06. $(3/8, 3\pi/16)$ 07. $(0, 45/(14\pi))$

08. 꼭짓점이 $(0, 0)$ 이고 변들이 양의 축을 따라 있을 때, $(2a/5, 2a/5)$

09. $I_x = \frac{64}{315}k, I_y = \frac{8}{105}k, I_0 = \frac{88}{315}k$

10. 꼭짓점이 $(0, 0)$ 이고 변들이 양의 축을 따라 있을 때,

$$I_x = 7ka^6/180, I_y = 7ka^6/180, I_0 = 7ka^6/90$$

11. $m = \frac{3\pi}{64}, (\bar{x}, \bar{y}) = \left(\frac{16384\sqrt{2}}{10395\pi}, 0\right), I_x = \frac{5\pi}{384} - \frac{4}{105}, I_y = \frac{5\pi}{384} + \frac{4}{105}, I_0 = \frac{5\pi}{192}$

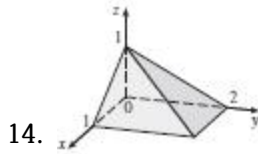
12. $I_x = \rho b h^3/3, I_y = \rho b^3 h/3; b/\sqrt{3}, h/\sqrt{3}$

연습문제 12.5

01. $\frac{27}{4}$ 02. $\frac{16}{15}$ 03. $-\frac{1}{3}$ 04. $\frac{27}{2}$ 05. $\frac{9}{8}\pi$

06. $\frac{65}{28}$ 07. $\frac{1}{60}$ 08. $\frac{16}{3}\pi$ 09. $\frac{16}{3}$ 10. $\frac{8}{15}$

11. (a) $\int_0^1 \int_0^x \int_0^{\sqrt{1-y^2}} dz dy dx$ (b) $\frac{1}{4}\pi - \frac{1}{3}$ 12. ≈ 239.64 13. 0.985



15.
$$\begin{aligned} & \int_{-2}^2 \int_0^{4-x^2} \int_{-\sqrt{4-x^2-y}/2}^{\sqrt{4-x^2-y}/2} f(x, y, z) dz dy dx \\ &= \int_0^4 \int_{-\sqrt{4-y}}^{\sqrt{4-y}} \int_{-\sqrt{4-x^2-y}/2}^{\sqrt{4-x^2-y}/2} f(x, y, z) dz dx dy \\ &= \int_{-1}^1 \int_0^{4-4z^2} \int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x, y, z) dx dy dz \\ &= \int_0^4 \int_{-\sqrt{4-y}/2}^{\sqrt{4-y}/2} \int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x, y, z) dx dz dy \end{aligned}$$

$$\begin{aligned}
&= \int_{-2}^2 \int_{-\sqrt{4-x^2}/2}^{\sqrt{4-x^2}/2} \int_0^{4-x^2-4z^2} f(x, y, z) dy dz dx \\
&= \int_{-1}^1 \int_{-\sqrt{4-4z^2}}^{\sqrt{4-4z^2}} \int_0^{4-x^2-4z^2} f(x, y, z) dy dx dz \\
16. \quad &\int_{-2}^2 \int_{x^2}^4 \int_0^{2-y/2} f(x, y, z) dz dy dx = \int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} \int_0^{2-y/2} f(x, y, z) dz dx dy \\
&= \int_0^2 \int_0^{4-2z} \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y, z) dx dy dz \\
&= \int_0^4 \int_0^{2-y/2} \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y, z) dx dz dy \\
&= \int_{-2}^2 \int_0^{2-x^2/2} \int_{x^2}^{4-2z} f(x, y, z) dy dz dx \\
&= \int_0^2 \int_{-\sqrt{4-2z}}^{\sqrt{4-2z}} \int_{x^2}^{4-2z} f(x, y, z) dy dx dz \\
17. \quad &\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x, y, z) dz dy dx = \int_0^1 \int_0^{y^2} \int_0^{1-y} f(x, y, z) dz dx dy \\
&= \int_0^1 \int_0^{1-z} \int_0^{y^2} f(x, y, z) dx dy dz \\
&= \int_0^1 \int_0^{1-y} \int_0^{y^2} f(x, y, z) dx dz dy \\
&= \int_0^1 \int_0^{1-\sqrt{x}} \int_{\sqrt{x}}^{1-z} f(x, y, z) dy dz dx \\
&= \int_0^1 \int_0^{(1-z)^2} \int_{\sqrt{x}}^{1-z} f(x, y, z) dy dx dz
\end{aligned}$$

$$\begin{aligned}
18. \quad \int_0^1 \int_y^1 \int_0^y f(x, y, z) dz dx dy &= \int_0^1 \int_0^x \int_0^y f(x, y, z) dz dy dx \\
&= \int_0^1 \int_z^1 \int_y^1 f(x, y, z) dx dy dz \\
&= \int_0^1 \int_0^y \int_y^1 f(x, y, z) dx dz dy \\
&= \int_0^1 \int_0^x \int_z^x f(x, y, z) dy dz dx \\
&= \int_0^1 \int_z^1 \int_z^x f(x, y, z) dy dx dz
\end{aligned}$$

$$19. 64\pi \quad 20. \frac{79}{30}, \left(\frac{358}{553}, \frac{33}{79}, \frac{571}{553} \right)$$

$$21. a^5, (7a/12, 7a/12, 7a/12)$$

$$22. (a) m = \int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} \sqrt{x^2 + y^2} dz dy dx$$

$$(b) (\bar{x}, \bar{y}, \bar{z}), \text{여기서}$$

$$\bar{x} = \frac{1}{m} \int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} x \sqrt{x^2 + y^2} dz dy dx,$$

$$\bar{y} = \frac{1}{m} \int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} y \sqrt{x^2 + y^2} dz dy dx,$$

$$\bar{z} = \frac{1}{m} \int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} z \sqrt{x^2 + y^2} dz dy dx$$

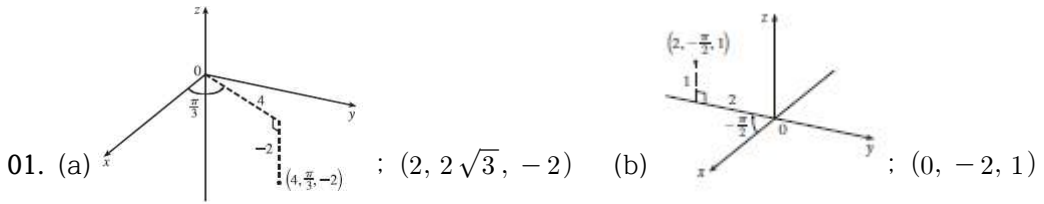
$$(c) I_z = \int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_0^{\sqrt{1-x^2-y^2}} x^2 + y^2 (1+x+y+z) dz dy dx$$

$$23. (a) \frac{3}{32}\pi + \frac{11}{24} \quad (b) \left(\frac{28}{9\pi+44}, \frac{30\pi+128}{45\pi+220}, \frac{45\pi+208}{135\pi+660} \right) \quad (c) \frac{1}{240}(68+15\pi)$$

$$24. I_x = I_y = I_z = \frac{2}{3}kL^5 \quad 25. \frac{1}{2}\pi kha^4 \quad 26. L^3/8$$

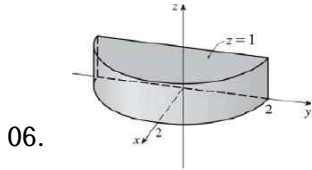
$$27. (a) \text{타원체 } x^2 + 2y^2 + 3z^2 = 1 \text{로 둘러싸인 영역} \quad (b) 4\sqrt{6}\pi/45$$

연습문제 12.6

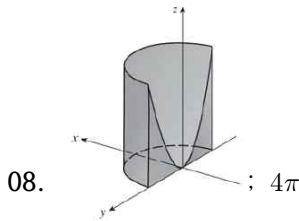


02. (a) $(\sqrt{2}, 3\pi/4, 1)$ (b) $(4, 2\pi/3, 3)$ 03. z 축을 지나는 수직 반평면

04. 원형 포물면 05. (a) $z^2 = 1 + r \cos \theta - r^2$ (b) $z = r^2 \cos 2\theta$



07. 원기둥좌표: $6 \leq r \leq 7, 0 \leq \theta \leq 2\pi, 0 \leq z \leq 20$

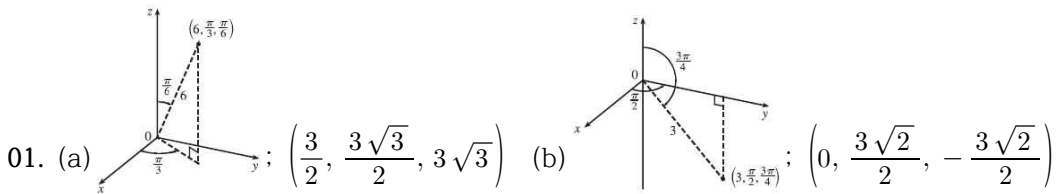


09. 384π 10. $\frac{8}{3}\pi + \frac{128}{15}$ 11. $\frac{2\pi}{5}$ 12. $\frac{4}{3}\pi(\sqrt{2}-1)$

13. (a) 162π (b) $(0, 0, 15)$ 14. $\pi K a^2/8, (0, 0, 2a/3)$ 15. 0

16. (a) $\iiint_C h(P)g(P)dV$, 여기서 C 는 원뿔이다. (b) $\approx 4.4 \times 10^8 \text{ J}$

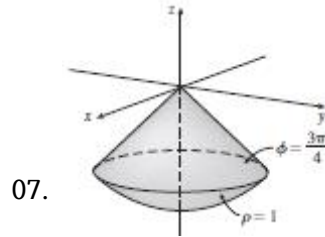
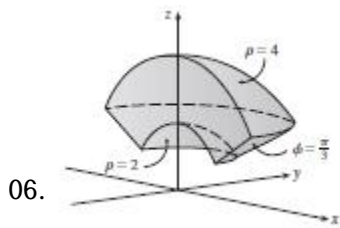
연습문제 12.7



02. (a) $(2, 3\pi/2, \pi/2)$ (b) $(2, 3\pi/4, 3\pi/4)$ 03. 원뿔의 상반부

04. 구, 반지름 $\frac{1}{2}$, 중심 $(0, \frac{1}{2}, 0)$

05. (a) $\cos^2 \phi = \sin^2 \phi$ (b) $\rho^2(\sin^2 \phi \cos^2 \theta + \cos^2 \phi) = 9$



08. $0 \leq \phi \leq \pi/4, 0 \leq \rho \leq \cos \phi$

09. ; $(9\pi/4)(2 - \sqrt{3})$

10. $\int_0^{\pi/2} \int_0^3 \int_0^2 f(r \cos \theta, r \sin \theta, z) r dz dr d\theta$

11. $\frac{312500\pi}{7}$

12. $\frac{1688\pi}{15}$

13. $\frac{\pi}{8}$

14. (a) 10π (b) $(0, 0, 2.1)$

15. (a) $\left(0, 0, \frac{7}{12}\right)$ (b) $11K\pi/960$

16. (a) $\left(0, 0, \frac{3}{8}a\right)$ (b) $4K\pi a^5/15$

17. $\frac{1}{3}\pi(2 - \sqrt{2}), \left(0, 0, \frac{3}{8(2 - \sqrt{2})}\right)$

18. $5\pi/6$

19. $(4\sqrt{2} - 5)/15$



20. $4096\pi/21$

21.

22. $136\pi/99$

23. (a)
$$\begin{aligned} V &= \int_0^{2\pi} \int_0^{\alpha \sin \phi_0} \int_{r \cot \phi_0}^{\sqrt{a^2 - r^2}} r dz dr d\theta \\ &= 2\pi \int_0^{\alpha \sin \phi_0} (r \sqrt{a^2 - r^2} - r^2 \cot \phi_0) dr \\ &= \frac{2\pi}{3} \left[-(a^2 - r^2)^{3/2} - r^3 \cot \phi_0 \right]_0^{\alpha \sin \phi_0} \\ &= \frac{2\pi}{3} \left[-(a^2 - a^2 \sin^2 \phi_0)^{3/2} - a^3 \sin^3 \phi_0 \cot \phi_0 + a^3 \right] \\ &= \frac{2}{3}\pi a^3 [1 - (\cos^3 \phi_0 + \sin^2 \phi_0 \cos \phi_0)] \\ &= \frac{2\pi a^3}{3} (1 - \cos \phi_0) \end{aligned}$$

(b)
$$\begin{aligned} \Delta V &= \frac{1}{3}(\theta_2 - \theta_1) [\rho_2^3(1 - \cos \phi_2) - \rho_2^3(1 - \cos \phi_1) - \rho_1^3(1 - \cos \phi_2) + \rho_1^3(1 - \cos \phi_1)] \\ &= \frac{1}{3}(\theta_2 - \theta_1) [(\rho_2^3 - \rho_1^3)(1 - \cos \phi_2) - (\rho_2^3 - \rho_1^3)(1 - \cos \phi_1)] \end{aligned}$$

$$= \frac{\rho_2^3 - \rho_1^3}{3} (\cos \phi_1 - \cos \phi_2) (\theta_2 - \theta_1)$$

(c) 생략

연습문제 12.8

01. 16 02. $\sin^2 \theta - \cos^2 \theta$ 03. 0
04. 꼭짓점이 $(0, 0)$, $(6, 3)$, $(12, 1)$, $(6, -2)$ 인 평행사변형
05. 직선 $y = 1$, y 축 및 $y = \sqrt{x}$ 로 둘러싸인 영역
06. 가능한 변환 중의 하나는 $x = \frac{1}{3}(v - u)$, $y = \frac{1}{3}(u + 2v)$ 이다. 여기서
 $S = \{(u, v) \mid -1 \leq u \leq 1, 1 \leq v \leq 3\}$ 이다.
07. 가능한 변환 중의 하나는 $x = u \cos v$, $y = u \sin v$ 이다. 여기서
 $S = \{(u, v) \mid 1 \leq u \leq \sqrt{2}, 0 \leq v \leq \pi/2\}$ 이다.
08. -3 09. 6π 10. $2 \ln 3$
11. (a) $\frac{4}{3}\pi abc$ (b) $1.083 \times 10^{12} \text{ km}^3$ (c) $\frac{4}{15}\pi(a^2 + b^2)abck$
12. $\frac{8}{5} \ln 8$ 13. $\frac{3}{2} \sin 1$ 14. $e - e^{-1}$

12장 복습문제

참-거짓 질문

01. 참 02. 참 03. 참 04. 참 05. 거짓

연습문제

01. 64.0 02. $4e^2 - 4e + 3$ 03. $\frac{1}{2} \sin 1$ 04. $\frac{2}{3}$
05. $\int_0^\pi \int_2^4 f(r \cos \theta, r \sin \theta) r dr d\theta$
06. 제1사분면에서 사엽장미 $r = \sin 2\theta$ 의 고리 내부 영역
07. $\frac{1}{2} \sin 1$ 08. $\frac{1}{2}e^6 - \frac{7}{2}$ 09. $\frac{1}{4} \ln 2$ 10. 8 11. $81\pi/5$
12. $\frac{81}{2}$ 13. $\pi/96$ 14. $\frac{64}{15}$ 15. 176 16. $\frac{2}{3}$ 17. $2ma^3/9$

$$18. \text{ (a) } \frac{1}{4} \quad \text{(b) } \left(\frac{1}{3}, \frac{8}{15} \right) \quad \text{(c) } I_x = \frac{1}{12}, I_y = \frac{1}{24}; \bar{\bar{y}} = \frac{1}{\sqrt{3}}, \bar{\bar{x}} = \frac{1}{\sqrt{6}}$$

$$19. \text{ (a) } (0, 0, h/4) \quad \text{(b) } \pi a^4 h/10 \quad 20. (\sqrt{3}, 3, 2), (4, \pi/3, \pi/3)$$

$$21. (2\sqrt{2}, 2\sqrt{2}, 4\sqrt{3}), (4, \pi/4, 4\sqrt{3}) \quad 22. r^2 + z^2 = 4, \rho = 2 \quad 23. 97.2$$

$$24. \int_0^1 \int_0^{1-z} \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y, z) dx dy dz \quad 25. -\ln 2 \quad 26. 0$$
