

연습문제 해답

게시 일자 : 2018-04-19

8장

급수

8.1 수열

01. $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13}$; 극한을 갖는다.; $\frac{1}{2}$

02. $a_n = (-1)^{n+1} \frac{n^2}{n+1}$

03. 수렴; 5

04. 발산

05. 발산

06. 수렴; 0

07. 수렴; $\ln 2$

08.

(a) 1060, 1123.60, 1191.02, 1262.48, 1338.23

(b) 발산

09. 감소, 유계수열

10. 단조수열이 아니다. 유계수열이 아니다.

11.

(a) 생략

(b) 2

12. $\frac{3-\sqrt{5}}{2}$

13. 62

14. 생략

15. 생략

8.2 급수

01.

(a) 수열은 수들의 순서에 의한 리스트인 반면에 급수는 수들의 리스트를 더한 것이다.

(b) 부분합이 수렴하면 급수는 수렴한다. 부분합이 수렴하지 않으면 급수는 발산한다.

02. 수렴; $\frac{1}{7}$

03. 발산

04. 발산

05. 발산

06. 수렴; $\frac{5}{2}$

07. 발산

08. 수렴; $\frac{11}{6}$

09. 16

10. $\frac{838}{333}$

11. $-3 < x < 3$; $\frac{x}{3-x}$

12. $-1 < x < 5$; $\frac{3}{5-x}$

13.

(a) 157.875mg; $\frac{3000}{19}(1 - 0.05^n)$ mg

(b) 157.895mg

14. 생략

15. 생략

16. 급수는 발산한다.

17.

(a) $\frac{1}{2}, \frac{5}{6}, \frac{23}{24}, \frac{119}{120}, \frac{(n+1)!-1}{(n+1)!}$

(b) 생략

(c) 1

8.3 적분 판정법과 비교 판정법

01.

(a) 수렴할 수도 있고 수렴하지 않을 수도 있다.

(b) 수렴한다.

02. 발산

03. 수렴

04. 발산

05. 수렴

06. 발산

07. 수렴

08. 수렴

09. 발산

10. $p > 1$

11. 1.037

12. 생략

13. 생략

14. 생략

15. 생략

16. 생략

17. 생략

8.4 다른 수렴 판정법들

01. 수렴

02. 발산

03. 5

04. -0.4597

05. 절대수렴

06. 조건부 수렴

07. 절대수렴

08. 절대수렴

09. 발산

10. 발산

11. 절대수렴

12. (a), (d)

13. 생략

8.5 거듭제곱급수

01. $R=1, (-1, 1)$

02. $R=\infty, (-\infty, \infty)$

03. $R=2, (-2, 2)$

04. $R=4, (-4, 4]$

05. $R=1, [1, 3]$

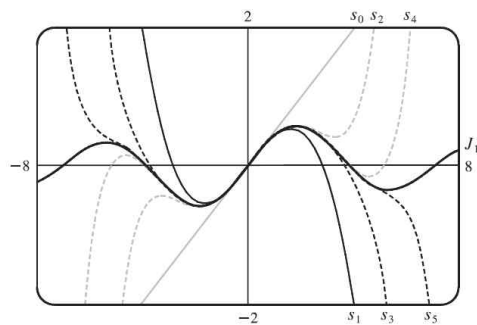
06. $R=0, \left\{ \frac{1}{2} \right\}$

07. k^k

08.

(a) $(-\infty, \infty)$

(b), (c)



09. $(-1, 1), f(x) = \frac{1+2x}{1-x^2}$

10. 2

8.6 함수를 거듭제곱급수로 표현하기

01. 10

02. $2 \sum_{n=0}^{\infty} \frac{1}{3^{n+1}} x^n, (-3, 3)$

03. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{9^{n+1}}, (-3, 3)$

04. $\sum_{n=0}^{\infty} \left[(-1)^{n+1} - \frac{1}{2^{n+1}} \right] x^n, (-1, 1)$

05.

(a) $\sum_{n=0}^{\infty} (-1)^n (n+1) x^n, R=1$

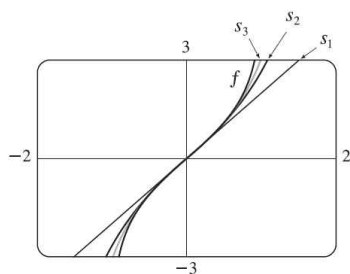
(b) $\frac{1}{2} \sum_{n=0}^{\infty} (-1)^n (n+2)(n+1) x^n$

(c) $\frac{1}{2} \sum_{n=2}^{\infty} (-1)^n (n)(n-1) x^n$

06. $\ln 5 - \sum_{n=1}^{\infty} \frac{x^n}{n 5^n}, R=5$

07. $\sum_{n=0}^{\infty} (2n+1) x^n, R=1$

08. $\sum_{n=0}^{\infty} \frac{2x^{2n+1}}{2n+1}$



09. $C + \sum_{n=0}^{\infty} \frac{t^{8n+2}}{8n+2}, \quad R=1$

10. $C + \sum_{n=1}^{\infty} (-1)^n \frac{x^{n+3}}{n(n+3)}, \quad R=1$

11. 0.199989

12. 0.000983

13. 0.19740

14. $[-1, 1], [-1, 1), (-1, 1)$

15. 생략

8.7 테일러 급수와 매클로린 급수

01. $\sum_{n=0}^{\infty} (n+1)x^n, R=1$

02. $\sum_{n=0}^{\infty} (n+1)x^n, R=1$

03. $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1}}{(2n+1)!} x^{2n+1}, R=\infty$

04. $\ln 2 + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n2^n} (x-2)^n$

05. $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{(x-\pi)^{2n}}{(2n)!}$

06. 생략

07. $1 - \frac{1}{4}x - \sum_{n=2}^{\infty} \frac{3 \cdot 7 \cdots (4n-5)}{4^n \cdot n!} x^n, R=1$

08. $\sum_{n=0}^{\infty} \frac{(-1)^n (n+1)(n+2)}{2^{n+4}} x^n, R=2$

09. $\sum_{n=0}^{\infty} \frac{2^n + 1}{n!} x^n$

10. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{2^{2n} (2n)!} x^{4n+1}$

11. $\frac{1}{2}x + \sum_{n=1}^{\infty} (-1)^n \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{n! 2^{3n+1}} x^{2n+1}$

12. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2^{2n-1} x^{2n}}{(2n)!}$

13.

(a) $1 + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n \cdot n!} x^{2n}$

(b) $x + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{(2n+1)2^n \cdot n!} x^{2n+1}$

14. $C + \sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+2}}{(6n+2)(2n)!} \quad , \quad R = \infty$

15. $C + \sum_{n=1}^{\infty} (-1)^n \frac{x^{2n}}{2n \cdot (2n)!} \quad , \quad R = \infty$

16. 0.440

17. $\frac{1}{2}$

18. $\frac{1}{120}$

19. $1 - \frac{3}{2}x^2 + \frac{25}{24}x^4 + \cdots$

20. e^{-x^4}

21. $\frac{1}{\sqrt{2}}$

22.

(a) $\sum_{n=1}^{\infty} nx^n$

(b) 2

23. 생략

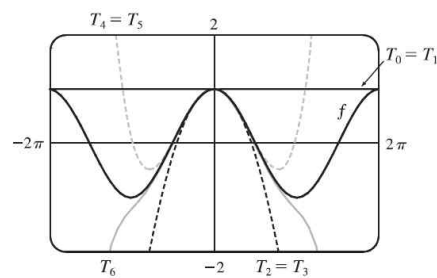
24. 생략

8.8 테일러 다항식의 응용

01.

(a)

n	$f^{(n)}(x)$	$f^{(n)}(0)$	$T_n(x)$
0	$\cos x$	1	1
1	$-\sin x$	0	1
2	$-\cos x$	-1	$1 - \frac{1}{2}x^2$
3	$\sin x$	0	$1 - \frac{1}{2}x^2$
4	$\cos x$	1	$1 - \frac{1}{2}x^2 + \frac{1}{24}x^4$
5	$-\sin x$	0	$1 - \frac{1}{2}x^2 + \frac{1}{24}x^4$
6	$-\cos x$	-1	$1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6$

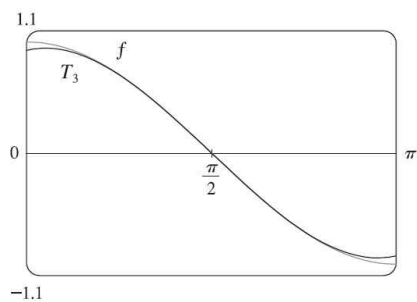


(b)

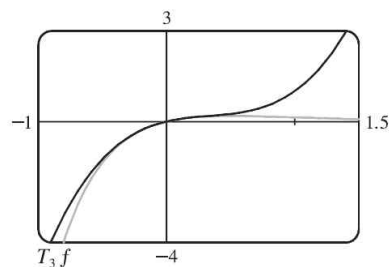
x	f	$T_0 = T_1$	$T_2 = T_3$	$T_4 = T_5$	T_6
$\frac{\pi}{4}$	0.7071	1	0.6916	0.7074	0.7071
$\frac{\pi}{2}$	0	1	-0.2337	0.0200	-0.0009
π	-1	1	-3.9348	0.1239	-1.2114

(c) 생략

02. $T_3(x) = -\left(x - \frac{\pi}{2}\right) + \frac{1}{6}\left(x - \frac{\pi}{2}\right)^3$



03. $T_3(x) = x - 2x^2 + 2x^3$

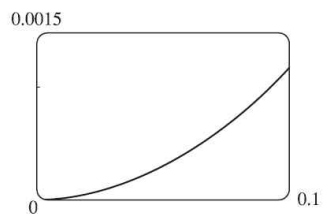


04.

(a) $1 + \frac{1}{2}x$

(b) 0.00125

(c)

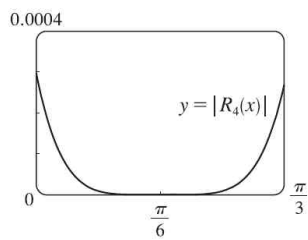


05.

(a) $\frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) - \frac{1}{4}\left(x - \frac{\pi}{6}\right)^2 - \frac{\sqrt{3}}{12}\left(x - \frac{\pi}{6}\right)^3 + \frac{1}{48}\left(x - \frac{\pi}{6}\right)^4$

(b) 0.000328

(c)

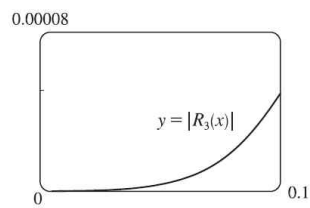


06.

(a) $1 + x^2$

(b) 0.00006

(c)



07. 0.17365

08. $-1.037 < x < 1.037$

09. 21m, 아니다.

10.

(a) 생략

(b) 생략

(c) 약 0.00000000808km 차이가 난다.

8장 복습문제

연습문제

01. 수렴: $\frac{1}{2}$

02. 수렴: 0

03. 수렴

04. 발산

05. 수렴

06. 수렴

07. 조건부 수렴

08. 절대수렴

09. 8

10. $\frac{\pi}{4}$

11. 생략

12. 생략

13. $R=4$, $[-6, 2)$

14. $R=\frac{1}{2}$, $[\frac{5}{2}, \frac{7}{2})$

15. $\frac{1}{2} \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!} \left(x - \frac{\pi}{6}\right)^{2n} + \frac{\sqrt{3}}{2} \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)!} \left(x - \frac{\pi}{6}\right)^{2n+1}$

16. $\sum_{n=0}^{\infty} (-1)^n x^{n+2}$, $R=1$

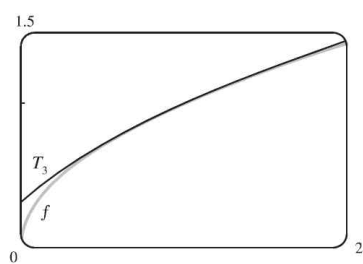
17. $\ln 4 - \sum_{n=1}^{\infty} \frac{x^n}{n4^n}, \quad R=4$

18. $C + \ln|x| + \sum_{n=1}^{\infty} \frac{x^n}{n \cdot n!}$

19.

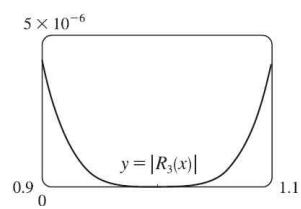
(a) $1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3$

(b)



(c) 0.000006

(d)



20. $-\frac{1}{6}$