

부록 E 해답

5장

연습문제 5.1

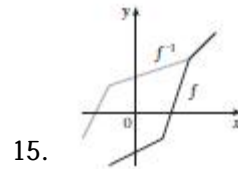
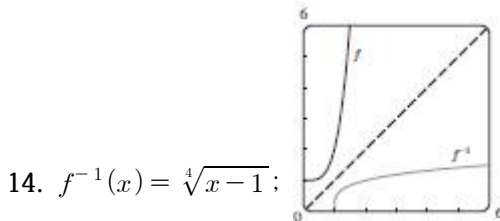
01. (a) 정의 1 참조. (b) 수평선 판정법을 통과해야 한다.
 02. 아니다. 03. 아니다. 04. 아니다. 05. 아니다. 06. 일대일이다.
 07. 아니다. 08. (a) 6 (b) 3 09. 4

10. $F = \frac{9}{5}C + 32$; 섭씨온도의 함수로써의 화씨온도: $[-273.15, \infty)$

11. $f^{-1}(x) = \frac{3}{2} - \frac{1}{2}x$

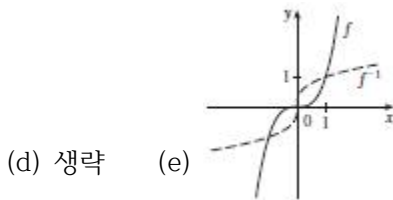
12. $f^{-1}(x) = \frac{1}{3}(x-1)^2 - \frac{2}{3}, x \geq 1$

13. $f^{-1}(x) = \left(\frac{1-x}{1+x}\right)^2, -1 < x \leq 1$

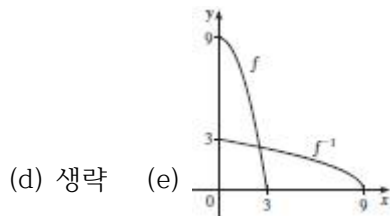


16. (a) $f^{-1}(x) = \sqrt{1-x^2}, 0 \leq x \leq 1$; f^{-1} 과 f 는 같은 함수이다.
 (b) 제1사분면에 있는 사분원

17. (a) 생략 (b) $\frac{1}{12}$ (c) $f^{-1}(x) = \sqrt[3]{x}$, 정의역 = \mathbb{R} = 치역



18. (a) 생략 (b) $-\frac{1}{2}$ (c) $f^{-1}(x) = \sqrt{9-x}$, 정의역 = $[0, 9]$, 치역 = $[0, 3]$



19. $\frac{1}{7}$

20. $\frac{2}{\pi}$

21. $\frac{3}{2}$

22. $\frac{1}{\sqrt{28}}$

$$23. f^{-1}(x) = -(\sqrt[3]{4}/6)(\sqrt[3]{D-27x^2+20} - \sqrt[3]{D-27x^2-20} + \sqrt[3]{2}),$$

여기서 $D = 3\sqrt{3}\sqrt{27x^4-40x^2+16}$; 두 가지 식이 복잡하다.

$$24. (a) g^{-1}(x) = f^{-1}(x) - c \quad (b) h^{-1}(x) = (1/c)f^{-1}(x)$$

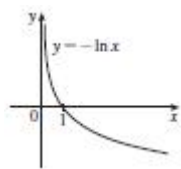
연습문제 5.2

$$01. \frac{1}{2} \ln a + \frac{1}{2} \ln b$$

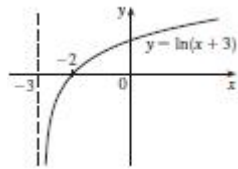
$$02. 2 \ln x - 3 \ln y - 4 \ln z$$

$$03. \ln 1215$$

$$04. \ln \frac{\sqrt{x}}{x+1}$$



05.



06.

$$07. -\infty$$

$$08. f'(x) = (2 + \ln x)/(2\sqrt{x})$$

$$09. f'(x) = \frac{\cos(\ln x)}{x}$$

$$10. f'(x) = -\frac{1}{x}$$

$$11. g'(x) = -\frac{2a}{a^2 - x^2}$$

$$12. G'(y) = \frac{10}{2y+1} - \frac{y}{y^2+1}$$

$$13. g'(x) = \frac{2x^2-1}{x(x^2-1)}$$

$$14. f'(u) = \frac{1+\ln 2}{u[1+\ln(2u)]^2}$$

$$15. y' = \frac{10x+1}{5x^2+x-2}$$

$$16. y' = \sec^2(\ln(ax+b)) \frac{a}{ax+b}$$

$$17. y' = x + 2x \ln(2x); y'' = 3 + 2 \ln(2x)$$

$$18. f'(x) = \frac{2x-1-(x-1)\ln(x-1)}{(x-1)[1-\ln(x-1)]^2}; (1, 1+e) \cup (1+e, \infty)$$

$$19. \frac{1}{2}$$

$$20. y = 2x - 2$$

$$21. y' = \frac{2x}{x^2+y^2-2y}$$

$$22. f^{(n)}(x) = \frac{(-1)^{n-1}(n-1)!}{(x-1)^n}$$

23. A. $(2n\pi, (2n+1)\pi)$ 안의 모든 x , 여기서 n 은 정수

B. x 절편 $\pi/2 + 2n\pi$

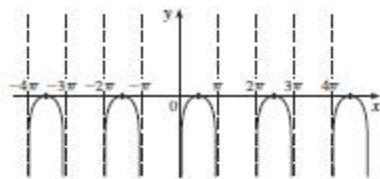
C. 2π 주기

D. 수직점근선 $x = n\pi$

E. 증가구간 $(2n\pi, \pi/2 + 2n\pi)$; 감소구간 $(\pi/2 + 2n\pi, (2n+1)\pi)$

F. 극대값 $f(\pi/2 + 2n\pi) = 0$

G. 아래로 오목 구간 $(2n\pi, (2n+1)\pi)$



H.

24. A. \mathbb{R}

B. y 절편 0, x 절편 0

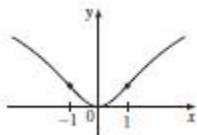
C. y 축 대칭

D. 없다.

E. 증가구간 $(0, \infty)$; 감소구간 $(-\infty, 0)$

F. 극솟값 $f(0) = 0$

G. 위로 오목 구간 $(-1, 1)$; 아래로 오목 구간 $(-\infty, -1), (1, \infty)$;
변곡점 $(\pm 1, \ln 2)$



H.

25. 증가구간 $(0, 2.7), (4.5, 8.2), (10.9, 14.3)$;

변곡점 $(3.8, 1.7), (5.7, 2.1), (10.0, 2.7), (12.0, 2.9)$

26. $y' = (x^2 + 2)^2 (x^4 + 4)^4 \left(\frac{4x}{x^2 + 2} + \frac{16x^3}{x^4 + 4} \right)$

27. $y' = \sqrt{\frac{x-1}{x^4+1}} \left(\frac{1}{2x-2} - \frac{2x^3}{x^4+1} \right)$

28. $\frac{1}{3} \ln \frac{5}{2}$

29. $\frac{1}{2} e^2 + e - \frac{1}{2}$

30. $\frac{1}{3} (\ln x)^3 + C$

31. $-\ln(1 + \cos^2 x) + C$

32. 생략

33. $\frac{1}{3}$

34. (a) 생략 (b) 0.405

35. 생략

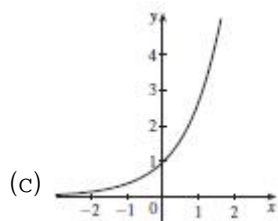
36. 생략

37. $0 < m < 1, m - 1 - \ln m$

38. 생략

연습문제 5.3

01. (a) e 는 $\ln e = 1$ 을 만족하는 수이다. (b) $e \approx 2.71828$



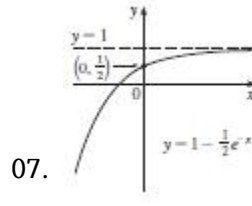
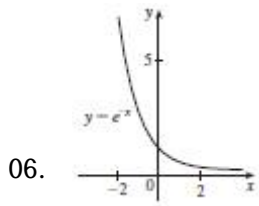
(c) ; $f(x) = e^x$ 이면 $f'(0) = 1$

02. (a) $\frac{1}{25}$ (b) 10

03. (a) $\frac{1}{4}(7 - \ln 6)$ (b) $\frac{1}{3}(e^2 + 10)$

04. (a) $\frac{1}{3}(\ln k - 1)$ (b) $\frac{1}{2}(1 + \sqrt{1 + 4e})$

05. (a) $0 < x < 1$ (b) $x > \ln 5$



08. (a) $\left(-\infty, \frac{1}{2}\ln 3\right]$ (b) $f^{-1}(x) = \frac{1}{2}\ln(3 - x^2)$, $[0, \sqrt{3})$

09. 1

10. 0

11. 0

12. $f'(x) = e^x(x^3 + 3x^2 + 2x + 2)$

13. $y' = 3ax^2e^{ax^3}$

14. $f'(u) = (-1/u^2)e^{1/u}$

15. $F'(t) = e^{t \sin 2t}(2t \cos 2t + \sin 2t)$

16. $y' = \frac{3e^{3x}}{\sqrt{1 + 2e^{3x}}}$

17. $y' = e^{e^x}e^x$

18. $y' = \frac{(ad - bc)e^x}{(ce^x + d)^2}$

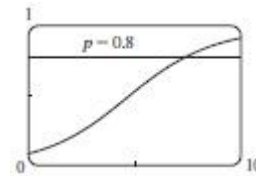
19. $y = 2x + 1$

20. $y' = \frac{y(y - e^{x/y})}{y^2 - xe^{x/y}}$

21. -4, -2

22. $f^{(n)}(x) = 2^n e^{2x}$

23. (a) 생략 (b) -0.567143



24. (a) 1 (b) $kae^{-kt}/(1 + ae^{-kt})^2$ (c) $t \approx 7.4$ h

25. -1

26. (a) 증가 구간 $(2, \infty)$; 감소 구간 $(-\infty, 2)$

(b) 위로 오목인 구간 $(-\infty, 3)$; 아래로 오목인 구간 $(3, \infty)$ (c) $(3, -2e^{-3})$

27. A. \mathbb{R}

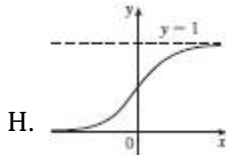
B. y 절편 $\frac{1}{2}$

C. 없다.

D. 수평점근선 $y = 0$, $y = 1$

E. 증가구간 \mathbb{R}

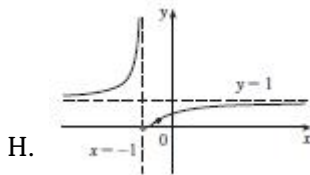
F. 없다.



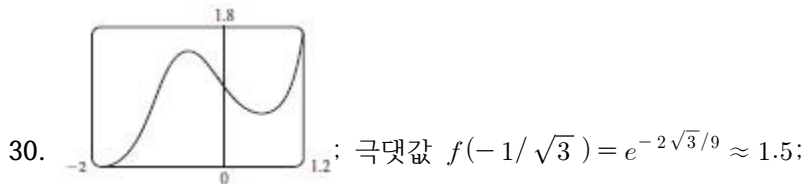
28. A. $\{x \mid x \neq -1\}$
 B. y 절편 $1/e$
 C. 없다.
 D. 수평점근선 $y=1$; 수직점근선 $x=-1$
 E. 증가구간 $(-\infty, -1) \cup (-1, \infty)$
 F. 없다.

G. 위로 오목 구간 $(-\infty, -1), (-1, -\frac{1}{2})$;

아래로 오목 구간 $(-\frac{1}{2}, \infty)$; 변곡점 $(-\frac{1}{2}, 1/e^2)$



29. 28.57분, 혈류 안의 약물수준의 증가율이 최대일 때; 85.71분, 감소율이 최대일 때



; 극댓값 $f(-1/\sqrt{3}) = e^{-2\sqrt{3}/9} \approx 1.5$;

극솟값 $f(1/\sqrt{3}) = e^{-2\sqrt{3}/9} \approx 0.7$; 변곡점 $(-0.15, 1.15), (-1.09, 0.82)$

31. $\frac{1}{\pi}(1 - e^{-2\pi})$

32. $\frac{2}{3}(1 + e^x)^{3/2} + C$

33. $e^{\tan x} + C$

34. $e - \sqrt{e}$

35. $\approx 4512L$

36. $\frac{1}{2}$

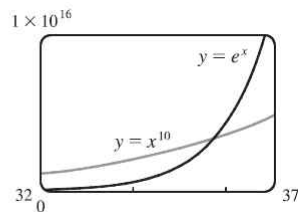
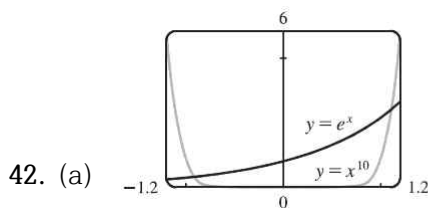
37. 생략

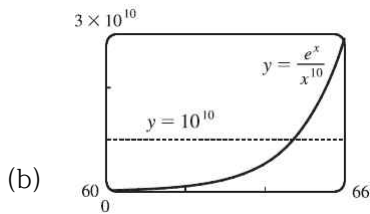
38. 생략

39. 생략

40. (a) 생략 (b) $\frac{43}{30} \leq \int_0^1 e^{x^2} dx \leq e - 1$.

41. 생략

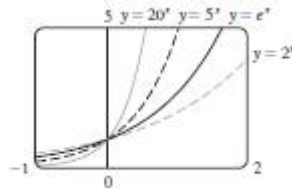




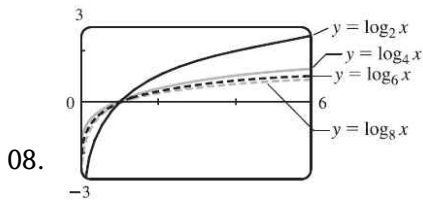
(c) $N \geq 65$

연습문제 5.4

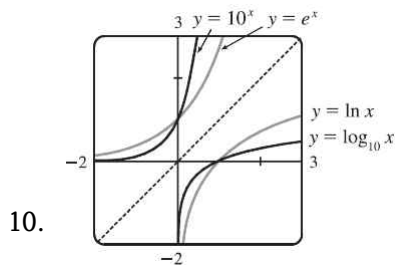
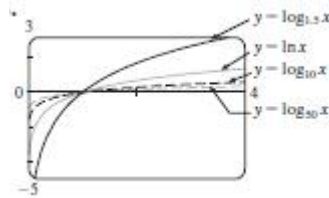
01. (a) $a^x = e^{x \ln a}$ (b) $(-\infty, \infty)$ (c) $(0, \infty)$ (d) [그림 23~25] 참조
 02. $e^{-\pi \ln 4}$ 03. $e^{x^2 \ln 10}$ 04. (a) 3 (b) -3 05. (a) 3 (b) -2
 06. $x \rightarrow -\infty$ 이면 모든 그래프가 0으로 접근한다. 모두 $(0, 1)$ 을 지나고 증가한다. 밑이 커질수록 증가율이 더 빠르다.



07. (a) 0.402430 (b) 1.454240 (c) 1.651496



09. $x \rightarrow 0^+$ 이면 모든 그래프가 $-\infty$ 로 접근한다. 모두 $(1, 0)$ 을 지나고 증가한다. 밑이 커질수록 증가율이 더 느리다.



11. $f(x) = 3 \cdot 2^x$ 12. (a) 생략 (b) 약 1084588 마일
 13. 0 14. $f'(x) = 5x^4 + 5^x \ln 5$ 15. $f'(t) = 10^{\sqrt{t}} \ln 10 / (2\sqrt{t})$
 16. $L'(v) = 2v \ln 4 \sec^2(4^{v^2}) \cdot 4^{v^2}$ 17. $y' = \frac{1}{\ln 10} + \log_{10} x$
 18. $y' = x^x (1 + \ln x)$ 19. $y' = x^{\sin x} \left(\frac{\sin x}{x} + \cos x \ln x \right)$
 20. $y' = (\cos x)^x (-x \tan x + \ln \cos x)$ 21. $y' = (\tan x)^{1/x} \left(\frac{\sec^2 x}{x \tan x} - \frac{\ln \tan x}{x^2} \right)$
 22. $y = (10 \ln 10)x + 10(1 - \ln 10)$ 23. $90/(\ln 10)$
 24. $(\ln x)^2 / (2 \ln 10) + C$ 또는 $\frac{1}{2} (\ln 10) (\log_{10} x)^2 + C$
 25. $3^{\sin \theta} / \ln 3 + C$ 26. $y = \log_{10} x - \log_{10} (1 - x)$ 27. 0
 28. 생략 29. 생략 30. 생략

연습문제 5.5

01. 약 235
 02. (a) $100(4.2)^t$ (b) ≈ 7409 (c) ≈ 10632 박테리아/h (d) $(\ln 100)/(\ln 42) \approx 3.2$ h
 03. (a) 1508백만 명, 1871백만 명 (b) 2161백만 명
 (c) 3972백만 명; 처음 반세기에 전쟁이 있었고, 후반부 반세기에 기대수명이 증가했다.
 04. (a) $Ce^{-0.0005t}$ (b) $-2000 \ln 0.9 \approx 211$ s
 05. (a) $100 \times 2^{-t/30}$ mg (b) ≈ 9.92 mg (c) ≈ 199.3 년
 06. ≈ 2500 년 07. (a) $\approx 58^\circ\text{C}$ (b) ≈ 98 분
 08. (a) $\approx 13.3^\circ\text{C}$ (b) ≈ 67.74 분
 09. (a) ≈ 64.5 kPa (b) ≈ 39.9 kPa
 10. (a) 3828.84달러 (b) 3840.25달러 (c) 3850.08달러 (d) 3851.61달러
 (e) 3852.01달러 (f) 3852.08달러

연습문제 5.6

01. (a) $\frac{\pi}{3}$ (b) π 02. (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{4}$ 03. (a) 10 (b) $\frac{\pi}{3}$ 04. 생략
 05. $x/\sqrt{1+x^2}$ 06.~09. 생략 10. $y' = \frac{2 \tan^{-1} x}{1+x^2}$
 11. $y' = \frac{1}{\sqrt{-x^2-x}}$ 12. $G'(x) = -1 - \frac{x \arccos x}{\sqrt{1-x^2}}$ 13. $h'(t) = 0$
 14. $y' = -\frac{\sin \theta}{1 + \cos^2 \theta}$ 15. $y' = \sin^{-1} x$ 16. $y' = \frac{\sqrt{a^2-b^2}}{a+b \cos x}$

17. $\pi/6$

18. $\pi/2$

19. $\frac{1}{4} \text{ rad/s}$

20. A에서 $5 - 2\sqrt{5}$ 떨어진 거리

21. $4\pi/3$

22. $\pi^2/72$

23. $\tan^{-1}x + \frac{1}{2} \ln(1+x^2) + C$

24. $\frac{1}{3} \sin^{-1}(t^3) + C$

25. $2 \tan^{-1} \sqrt{x} + C$

26. 생략

연습문제 5.7

01. (a) 0 (b) 1

02. (a) $\frac{3}{4}$ (b) $\frac{1}{2}(e^2 - e^{-2}) \approx 3.62686$

03. (a) 1 (b) 0

04.~12. 생략

13. $\operatorname{sech} x = \frac{3}{5}$, $\sinh x = \frac{4}{3}$, $\operatorname{csch} x = \frac{3}{4}$, $\tanh x = \frac{4}{5}$, $\coth x = \frac{5}{4}$

14. (a) 1 (b) -1 (c) ∞ (d) $-\infty$ (e) 0 (f) 1 (g) ∞ (h) $-\infty$ (i) 0

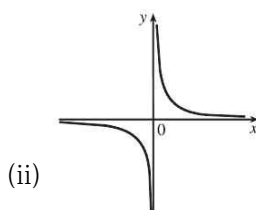
15. 생략

16. 생략

17. 생략

18. 생략

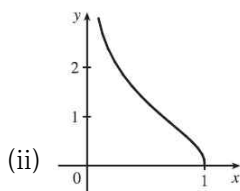
19. (a) (i) $y = \operatorname{csch}^{-1} x \Leftrightarrow \operatorname{csch} y = x \quad (x \neq 0)$



(ii)

(iii) $\operatorname{csch}^{-1} x = \ln \left(\frac{1}{x} + \frac{\sqrt{x^2 + 1}}{|x|} \right)$

(b) (i) $y = \operatorname{sech}^{-1} x \Leftrightarrow \operatorname{sech} y = x \text{ and } y > 0.$



(ii)

(iii) $\operatorname{sech}^{-1} x = \ln \left(\frac{1 + \sqrt{1 - x^2}}{x} \right)$

20. 생략

21. $f'(x) = x \cosh x$

22. $h'(x) = \tanh x$

23. $y' = 3e^{\cosh 3x} \sinh 3x$

24. 생략

25. $f'(t) = -2e^t \operatorname{sech}^2(e^t) \tanh(e^t)$

26. $G'(x) = \frac{-2 \sinh x}{(1 + \cosh x)^2}$

27. $y' = \frac{1}{2\sqrt{x(x-1)}}$

28. $y' = \sinh^{-1}(x/3)$

29. $y' = -\csc x$

30.~31. 생략

32. (a) $\sinh \frac{7}{20} \approx 0.3572$ (b) $\approx 70.34^\circ$

33. 생략

34. (a) 1176 N; $60 \cosh 1.6 \approx 164.50 \text{ m}$ (b) 120 m; $1200 \cosh 1.2 \approx 164.13 \text{ m}$

35. (a) 생략 (b) $y = 2 \sinh 3x - 4 \cosh 3x$ 36. $\frac{1}{3} \cosh^3 x + C$

37. $2 \cosh \sqrt{x} + C$ 38. $-\operatorname{csch} x + C$ 39. $\ln \left(\frac{6+3\sqrt{3}}{4+\sqrt{7}} \right)$

40. $\tanh^{-1}(e^x) + C$ 41. $(\ln(1+\sqrt{2}), \sqrt{2})$

연습문제 5.8

01. 2 02. $-\infty$ 03. 2 04. $\frac{1}{4}$ 05. $-\infty$ 06. $\frac{8}{5}$ 07. $\frac{1}{2}$

08. $1/\ln 3$ 09. $-\frac{1}{\pi^2}$ 10. $\frac{1}{2}a(a-1)$ 11. $\frac{1}{24}$ 12. 3 13. 0

14. $-2/\pi$ 15. $\frac{1}{2}$ 16. ∞ 17. 1 18. e^{-2} 19. $1/e$ 20. e^2

21. 생략 22. 생략 23. 1 24. 생략 25. 생략

26. $\frac{16}{9}a$ 27. $\frac{1}{2}$ 28. 생략 29. 56

30. 생략 31. (a) 0 (b) 생략

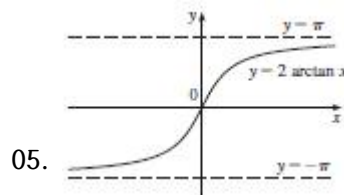
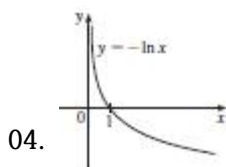
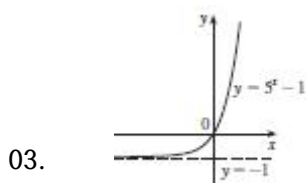
5장 복습문제

참-거짓 질문

01. 참 02. 거짓 03. 참 04. 참 05. 거짓 06. 거짓 07. 거짓
08. 참

연습문제

01. 아니다. 02. (a) 7 (b) $\frac{1}{8}$



06. (a) 9 (b) 2 07. (a) $\ln 5$ (b) e^2 08. (a) $\sqrt{1+e}$ (b) $\frac{d \ln 5}{\ln c}$

09. $y' = \frac{1 + \ln x}{x \ln x}$ 10. $y' = -\frac{e^{1/x}(1+2x)}{x^4}$ 11. $y' = \frac{1}{2\sqrt{\arctan x}(1+x^2)}$

12. $f'(t) = t + 2t \ln t$

13. $y' = 3^{x \ln x} (\ln 3)(1 + \ln x)$

14. $y' = 2x^2 \cosh(x^2) + \sinh(x^2)$

15. $h'(\theta) = 2 \sec^2(2\theta) e^{\tan 2\theta}$

16. $y' = \cot x - \sin x \cos x$
17. $y' = \frac{2}{(1+2x)\ln 5}$
18. $y' = \frac{(x-2)^4(3x^2-55x-52)}{2\sqrt{x+1}(x+3)^8}$
19. $y' = \frac{4x}{1+16x^2} + \tan^{-1}(4x)$
20. $y' = 3 \tanh 3x$
21. $y' = \frac{\cosh x}{\sqrt{\sinh^2 x - 1}}$
22. $y' = \frac{-3 \sin(e^{\sqrt{\tan 3x}}) e^{\sqrt{\tan 3x}} \sec^2(3x)}{2\sqrt{\tan 3x}}$
23. $f'(x) = g'(x) e^{g(x)}$
24. $f'(x) = \frac{g'(x)}{g(x)}$
25. $2^x (\ln 2)^n$
26. 생략
27. $(-3, 0)$
28. (a) $y = \frac{1}{4}x + \frac{1}{4}(\ln 4 + 1)$ (b) $y = e^x$
29. (a) $200(3.24)^t$ (b) ≈ 22040 (c) ≈ 25910 박테리아/h
(d) $(\ln 50)/(\ln 3.24) \approx 3.33$ h
30. (a) $C_0 e^{-kt}$ (b) ≈ 100 h
31. $\pi/2$
32. 0
33. $-\infty$
34. -1
35. 1
36. 8
37. 0
38. $\frac{1}{2}$
40. $-(1/x) - 2 \ln|x| + x + C$
41. $\frac{1}{4}(1 - e^{-2})$
42. $\arctan e - \pi/4$
43. $2e^{\sqrt{x}} + C$
44. $\frac{1}{2} \ln|x^2 + 2x| + C$
45. $-\frac{1}{2}[\ln(\cos x)]^2 + C$
46. $\frac{2}{3}$
47. $2/e$
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